

Modern meson–exchange potential and superfluid neutron star crust matter

Ø. Elgarøy, L. Engvik and E. Osnes

Department of Physics, University of Oslo, N-0316 Oslo, Norway

F. V. De Blasio, M. Hjorth–Jensen and G. Lazzari

ECT, European Centre for Theoretical Studies in Nuclear Physics and Related Areas,
Strada delle Tabarelle 286, I-38050 Villazzano (Trento), Italy*

In this work we study properties of neutron star crusts, where matter is expected to consist of nuclei surrounded by superfluid neutrons and a homogeneous background of relativistic electrons. The nuclei are disposed in a Coulomb lattice, and it is believed that the structure of the lattice influences considerably the specific heat of the neutronic matter inside the crust of a neutron star. Using a modern meson–exchange potential in the framework of a local–density approximation we calculate the neutronic specific heat accounting for various shapes of the Coulomb lattice, from spherical to non–spherical nuclear shapes. We find that a realistic nucleon–nucleon potential leads to a significant increase in the neutronic specific heat with respect to that obtained assuming a uniform neutron distribution. The increase is largest for the non–spherical phase of the crust. These results may have consequences for the thermal history of young neutron stars.

PACS number(s): 97.60.Jd 21.65.+f 74.25.Bt

The observation of thermal emission from the surface of a neutron star is a powerful tool by which one can obtain information about the state of matter inside the star. It has been shown that the time needed for a temperature drop in the core to affect the surface temperature should depend on the thickness of the crust and on its thermal properties, such as the total specific heat [1], which is strongly influenced by the superfluid state of matter inside the crust.

It has recently been proposed that the Coulomb-lattice structure of a neutron star crust may influence significantly the thermodynamical properties of the superfluid neutron gas [2]. The authors of Refs. [3–6] have proposed that in the crust of a neutron star non-spherical nuclear shapes could be present at densities ranging from $\rho = 1.0 \times 10^{14} \text{ gcm}^{-3}$ to $\rho = 1.5 \times 10^{14} \text{ gcm}^{-3}$, a density region which represents about 20% of the whole crust. The saturation density of nuclear matter is $\rho_0 = 2.8 \times 10^{14} \text{ gcm}^{-3}$. These unusual shapes are supposed [4,5] to be disposed in a Coulomb lattice embedded in an almost uniform background of relativistic electrons. According to the fact that the neutron drip point is supposed to occur at lower density ($\rho \sim 4.3 \times 10^{11} \text{ gcm}^{-3}$), and considering the characteristics of the nuclear force in this density range, we expect these unusual nuclear shapes to be surrounded by a gas of superfluid neutrons.

In the present paper we follow essentially Ref. [2], however, we differ in using one of the modern meson-exchange potentials of the Bonn group [7] in evaluating the neutron pairing energy gap. This potential is expected to give, contrary to the effective one used in Ref. [2], a more realistic estimate of the gaps in the region inside the nuclear cluster.

In the following we treat the crust-lattice in the Wigner-Seitz approximation, dividing the Coulomb lattice into unit cells of appropriate shape (cylindrical, planar and spherical), containing a nucleus surrounded by a gas of superfluid neutrons. Details of this system have been worked out within the framework of Thomas-Fermi calculations with different energy density functionals [3]. The neutron and proton density profiles are obtained from a recent parametrization of Oyamatsu [3]

$$\rho_i(r) = (\rho_i^{in} - \rho_i^{out}) \left[1 - \left(\frac{r}{R_i} \right)^{t_i} \right]^{k_i} + \rho_i^{out}, \quad (1)$$

for $r < R_i$ and

$$\rho_i(r) = \rho_i^{out}, \quad (2)$$

for $r > R_i$, where $i = p, n$ represent protons and neutrons respectively, with ρ_p^{out} taken to be zero. Following Oyamatsu [3] the spherical shape is expected for densities $\rho < 0.35\rho_0$ (about 80% of the whole crust). The cylindrical region is supposed to appear for densities ρ between $0.35\rho_0$ and $0.46\rho_0$, while in the region $0.46\rho_0 < \rho < 0.5\rho_0$ one presumes to have a slab-like form for the Coulomb lattice. The parameters R_i represent the finite bound-

ary of the nucleus and t_i determines the relative surface thickness.

With these density profiles we already have a fixed proton fraction relevant for the calculation of the pairing gap. Our calculation of the pairing gap is a two-step process (for details, see Ref. [8]). First we solve self-consistently the Brueckner-Hartree-Fock (BHF) equations for the single-particle energies, using a G -matrix defined through the Bethe-Brueckner-Goldstone equation as

$$G = V + V \frac{Q}{\omega - H_0} G, \quad (3)$$

where V is the nucleon-nucleon potential, Q is the Pauli operator which prevents scattering into intermediate states prohibited by the Pauli principle, H_0 is the unperturbed hamiltonian acting on the intermediate states and ω is the so-called starting energy, the unperturbed energy of the interacting states. Methods to solve this equation are reviewed in Ref. [9]. The single-particle energies for state k_i (i encompasses all relevant quantum numbers like momentum, isospin projection, angular momentum, spin etc.) in nuclear matter are assumed to have the simple quadratic form¹

$$\varepsilon_{k_i} = \frac{k_i^2}{2m^*} + \delta_i, \quad (4)$$

where m^* is the effective mass. The terms m^* and δ , the latter being an effective single-particle potential related to the G -matrix, are obtained through the self-consistent BHF procedure. The so-called model-space BHF method for the single-particle spectrum has been used, see e.g. Refs. [9,10], with a cutoff $k_M = 3.0 \text{ fm}^{-1}$. This self-consistency scheme consists in choosing adequate initial values of the effective mass and δ . The obtained G -matrix is in turn used to obtain new values for m^* and δ . This procedure continues until these parameters vary little. The BHF equations are solved for different proton fractions, using the formalism of Refs. [9,11]. The nucleon-nucleon potential is defined by the parameters of the meson-exchange potential model of the Bonn group, version A in Table A.2 of Ref. [7].

The next step is to evaluate the gap equation following the scheme proposed by Anderson and Morel [12] and applied to nuclear physics by Baldo *et al.* [13]. These authors introduced an effective interaction $\tilde{V}_{k,k'}$. This effective interaction sums up all two-particle excitations above the cutoff k_M . It is defined according to

$$\tilde{V}_{k,k'} = V_{k,k'} - \sum_{k'' > k_M} V_{k,k''} \frac{1}{2E_{k''}} \tilde{V}_{k'',k'}, \quad (5)$$

where the quasiparticle energy E_k is given by

¹We set $\hbar = c = 1$.

$$E_k = \sqrt{(\varepsilon_k - \varepsilon_F)^2 + \Delta_k^2}, \quad (6)$$

ε_F being the single-particle energy at the Fermi surface, $V_{k,k'}$ is the free nucleon-nucleon potential in momentum space and Δ_k is the pairing gap

$$\Delta_k = - \sum_{k' \leq k_M} \tilde{V}_{k,k'} \frac{\Delta_{k'}}{2E_{k'}}. \quad (7)$$

For notational economy, we have dropped the subscript i on the single-particle energies.

In summary, first we obtain the self-consistent BHF single-particle spectrum ε_k , thereafter we solve self-consistently Eqs. (5) and (7) in order to obtain the pairing gap Δ . This pairing gap is again calculated for various proton fractions according to Eq. (1). For states above k_M , the quasiparticle energy of (6) is approximated by $E_k = (\varepsilon_k - \varepsilon_F)$, an approximation found to yield satisfactory results in neutron matter [8].

FIG. 2. Neutron effective mass ratio m^*/m as function of the position r inside the Wigner-Seitz cell for the different densities and shapes reported in Fig. 1

FIG. 1. Neutron pairing energy gap Δ as function of the position r inside the Wigner-Seitz cell for various nuclear shapes and densities. For the spherical phase, $\rho/\rho_0 = 0.058$ and $\rho/\rho_0 = 0.176$ we have used solid and dashed lines, respectively. The cylindrical region ($\rho/\rho_0 = 0.354$) is given by a dotted line, while the slab region ($\rho/\rho_0 = 0.48$) is shown with a dash-dotted line.

The results for the neutron pairing gap, effective mass and local Fermi momentum as a function of the position in the Wigner-Seitz cell are displayed in Figs. 1, 2 and 3, respectively, for various density regions inside the crust.

FIG. 3. Neutron Fermi momentum as function of the position r inside the Wigner-Seitz cell for various nuclear shapes and densities as reported in Fig. 1

For heavy and medium heavy nuclei present in the neutron star crust, we expect mean radii of the order $6 \sim 8$ fm. We notice in Fig. 3 that the Fermi momentum for $r < 6$ fm is close to that of the saturation density of nuclear matter, or the central density of ^{208}Pb . At these densities, the gap energy in nuclear matter is generally small [8,13]. This is also seen in Fig. 1, where the gap is less than 0.5 MeV. The differences between the gap energies for the various shapes of the Coulomb lattice

can be retraced to the different effective masses in Fig. 2, since the effective masses which enter the determination of the pairing gap, differ. For values of r between 6 and 8 fm, close to the Fermi surface, where the Fermi momentum of Fig. 3 changes rapidly, we see the largest variations in the effective mass and the pairing gap, the latter reaching a peak of 2.5 MeV for the spherical phase. For larger values of r , i.e. outside the nucleus, the Fermi momentum stabilizes, though there are large differences from shape to shape. For the spherical shape given by the lowest density $\rho/\rho_0 = 0.058$ we get the lowest value of k_F , whereas for the slab phase with $\rho/\rho_0 = 0.48$ (half nuclear matter saturation density), we have $k_F \approx 1.3 \text{ fm}^{-1}$. Since the neutron 1S_0 pairing gap reaches its maximum at k_F between $0.8 \sim 1.0 \text{ fm}^{-1}$ [8,13], we see from Fig. 1, that the pairing gap is at its largest for the spherical phases and for radii larger than 8 fm. For large values of r , the pairing gaps are constant, since we have uniform neutron matter at a fixed k_F .

The qualitative features exhibited in Figs. 1–3, are similar to the results of Broglia *et al.* [2]. However, the pairing gaps obtained with the effective interaction of Ref. [2], are larger than those obtained here. Broglia *et al.* obtain maximum pairing gaps of the order of 3.5 MeV, whereas ours are of the order of 2.5 MeV. Our pairing gap for uniform matter is close to that of Baldo *et al.* [13], who also employ realistic nucleon–nucleon potentials. Moreover, in applications to finite nuclei [9], the nucleon–nucleon force used here reproduces very well e.g. the experimental spacing between the 0^+ ground state and the first excited 2^+ state of the tin isotopes. The smaller energy gap in this work may in turn have important consequences for thermal properties of neutron stars. To see this, we evaluate the specific heat for a system of superfluid neutrons. Here we use the thermodynamical expression

$$C_V = T \left(\frac{\partial S}{\partial T} \right)_V \quad (8)$$

where V is the volume of the system, T is the temperature and S is the total entropy. We obtain then the following form for the specific heat (see Ref. [2] for further details)

$$C_{Vn}^{sup} = \frac{k_B}{V_{WS}} \frac{1}{\pi T} \int_{V_{sup}} dr r^2 \int dk k^2 \frac{E_{k_F}(r)}{\cosh^2 \left(\frac{E_{k_F}(r)}{2T} \right)} \times \left(\frac{E_{k_F}(r)}{T} - \frac{dE_{k_F}(r)}{dT} \right), \quad (9)$$

where $E_{k_F}(r)$ is the local quasi-particle energy

$$E_{k_F}(r) = \sqrt{(\epsilon_k(r) - \epsilon_F)^2 + \Delta_{k_F}^2(r)}, \quad (10)$$

V_{WS} is the volume of the Wigner–Seitz cell and V_{sup} is the volume occupied by the superfluid. With increasing

temperature the system of superfluid neutrons exhibits a phase transition towards a normal Fermi liquid system. The contribution to the total neutronic specific heat inside the Wigner–Seitz cell is written as

$$C_{Vn}^{norm} = \frac{k_B}{V_{WS}} \frac{1}{\pi T^2} \int_{V_{norm}} dr r^2 \int dk k^2 \times \frac{(\epsilon_k(r) - \epsilon_F)^2}{\cosh^2 \left((\epsilon_k(r) - \epsilon_F)/2T \right)}, \quad (11)$$

where V_{norm} is the volume occupied by the normal system inside the cell.

The total neutronic specific heat can be written for all temperatures as

$$C_n = C_{Vn}^{sup} + C_{Vn}^{norm}. \quad (12)$$

In order to show the relevance of our results, we compute the total fermionic specific heat given by

$$C_T = C_n + C_e \quad (13)$$

where C_e is the specific heat for relativistic electrons

$$C_e = \pi k_B \frac{T}{\epsilon_F}, \quad (14)$$

and ϵ_F is the Fermi energy of the electrons [14]. The large anisotropy found in the local pairing energy gap (see Fig. 1), leads to a larger specific heat for the superfluid neutron gas with respect to that obtained for the uniform neutron system. This is due to the weak superfluid neutron component inside the nucleus, see Fig. 1. This effect yields a larger specific heat compared to that of the superfluid neutron matter outside the nucleus [15].

FIG. 4. Ratio C_T^{nu}/C_T^u as function of temperature T for spherical and non-spherical phases inside the crust. Notations as in Fig. 1

In Fig. 4 we compare the ratio $C_T(n.u.)/C_T(u.)$ between the total fermionic specific heat evaluated accounting for non-uniform nuclear shapes and that obtained considering uniform neutron matter only. We notice that the ratio increases dramatically moving from spherical to plate-like nuclei, and for $T > 0.008$ MeV, typical of the inner crust of a neutron star in the first 10^2 yr after formation, the non-spherical nuclear phases give specific heats which are much larger than that for uniform neutron matter. This strong increase is due to the corresponding increase in the nuclear volume observed for non-spherical phases [16]. This increase overcomes the reduction in the anisotropy obtained in the pairing energy gap moving from spheres to slabs.

In summary, our results may have consequences for the thermal evolution of the star compared to models where the crust is described in terms of uniform neutron matter only. In fact properties like the heat diffusion time through the crust [17] may be affected. Moreover, compared to the results reported by Broglia *et al.* [2], where the Gogny interaction was employed to obtain the pairing gap, there is a further enhancement of the ratio $C_T(n.u.)/C_T(u.)$.

This work has been supported by the Istituto Trentino di Cultura, Italy, the Research Council of Norway and the NorFA (Nordic Academy for Advanced Research).

(1995).

- [17] F. V. De Blasio, G. Lazzari, P. M. Pizzochero and R. A. Broglia, submitted to Phys. Rev. D.

-
- [1] G. E. Brown, K. Kubodera, D. Page and P. Pizzochero, Phys. Rev. D **37**, 2042 (1988).
[2] R. A. Broglia, F. De Blasio, G. Lazzari, M. C. Lazzari and P. M. Pizzochero, Phys. Rev. D **50**, 4781 (1994).
[3] K. Oyamatsu, Nucl. Phys. **A561**, 431 (1993).
[4] C. P. Lorenz, D. G. Ravenhall and C. J. Pethick, Phys. Rev. Lett. **70**, 379 (1993).
[5] C. J. Pethick, D. G. Ravenhall and C. P. Lorenz, Nucl. Phys. **A585**, 675 (1995).
[6] C. J. Pethick and D. G. Ravenhall, University of Illinois report P-95-05-022, to be published.
[7] R. Machleidt, Adv. Nucl. Phys. **19**, 185 (1989).
[8] Ø. Elgarøy, MSc. Thesis, University of Oslo (1995), unpublished.
[9] M. Hjorth-Jensen, T. T. S. Kuo and E. Osnes, Phys. Reports, in press.
[10] T. T. S. Kuo and Z. Y. Ma, Phys. Lett. **B127**, 123 (1983).
[11] H. Q. Song, Z. X. Wang and T. T. S. Kuo, Phys. Rev. C **46**, 1788 (1992).
[12] P. W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961).
[13] M. Baldo, J. Cugnon, A. Lejeune and U. Lombardo, Nucl. Phys. **A515**, 409 (1990).
[14] G. Baym, H. Bethe and C. J. Pethick, Nucl. Phys. **A175**, 225 (1971).
[15] G. Lazzari, F. V. De Blasio, Z. Phys. **A349** (1994) 7.
[16] F. V. De Blasio and G. Lazzari, Phys. Rev. **C52**, 418







